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The TASTE Test

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DE GUSTIBUS NON EST DISPUTANDUM.[2]
Roman Proverb

The TASTE test, which has been an important component of spacecraft attitude mission support for more than a quarter-century, is documented here. The TASTE test permitted data validation and editing for direction sensors to be automated for the first time, greatly decreasing data processing time, and was an important reason for the rapid adoption of QUEST. The statistical properties of the TASTE test are derived. The value of the TASTE test for implementation in modern CCD star trackers is presented.

INTRODUCTION

In the late 1970s, there was a great concern at NASA Goddard Space Flight Center that the increasingly more demanding requirements in estimation accuracy and computational frequency would soon overwhelm the data-processing capacity for attitude support and create long delays in data processing. This was particularly true for the Magsat mission, scheduled for launch in October 1979, which would have the most demanding requirements to date. It is for this reason that the QUEST algorithm [1] was developed.[3]. It is not generally recognized, however, that from a mission perspective the most important aspect of QUEST for data processing was not the lightning speed of its

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[2]Concerning taste, there is naught to dispute.

[3]The story of the development of the QUEST algorithm, with reference also to the present topic has been told with some irreverence in reference [2]. The QUEST algorithm in 1979 was the fastest algorithm for batch optimal least-square attitude estimation, a position which it has held to this date, although QUEST now shares it now with other algorithms [3]

attitude computation but the efficiency with which it could verify the input data with a single simple scalar test. This was the TASTE test.

Before QUEST, data validation was carried out by fitting a curve to the data, removing outliers by hand, and then refitting the curve to the remaining data in order to smooth and interpolate the attitude estimates. Such a procedure required the continuous and costly intervention of the analyst. For the Magsat mission, it was anticipated that processing a single day of data, comprising, perhaps, more than 300,000 attitude estimates, might require several days, so that the eventually eight-month Magsat mission might require several years of data processing. With QUEST and the TASTE test, one day of data could be processed in only four hours, almost totally automatically. Not surprisingly, this led to the almost universal adoption of QUEST for both near-Earth and deep-space missions within a few years.

Surprisingly, the TASTE test has never been documented archivally. It appears as a comment in the FORTRAN code for the Magsat ground support software in 1979 and in an internal report in 1993. The result has been cited by Markley and Mortari (with attribution to this writer) in 2000 [4].[4] It was finally published in the open literature in 2005 [6].

THE WAHBA PROBLEM AND QUEST

Most attitude determination systems, especially those employing CCD star-trackers, use an attitude estimation algorithm based on the Wahba problem [7], which minimizes the least-square loss function

$$L(A) = \frac{1}{2}\sum_{k=1}^{N} a_k\,|\hat{\mathsf{W}}_k' - A\hat{\mathsf{V}}_k|^2 \tag{1}$$

with attitude estimate[5] $A^{*\prime}$ given by

$$A^{*\prime} = \arg\min_{A\epsilon SO(3)} L(A) \tag{2}$$

that is, the value of the 3×3 proper orthogonal matrix A for which $L(A)$ is a minimum. Here the a_k, $k = 1, \ldots, N$, are a set of N non-negative weights, $\hat{\mathsf{W}}_k'$, $k = 1, \ldots, N$, are the observed directions in the spacecraft body frame, and $\hat{\mathsf{V}}_k$, $k = 1, \ldots, N$, are the corresponding vectors in the primary reference frame (typically inertial). Generally, one assumes that the reference directions are noise-free.

A number of algorithms were proposed almost immediately for solving the Wahba problem. These and more recent approaches have been described by Markley and Mortari in their excellent review [4].[6] Of special importance have been Davenport's q-algorithm [1, 9] (developed in 1977) and QUEST [1], which has received wide

[4]Readers should be wary of the comparisons made of QUEST with the algorithms developed by the authors of Reference [4]. A more complete and more balanced comparison is presented in References [3] and [5].

[5]The $\hat{\mathsf{W}}_k'$ are the realizations of a random vector $\hat{\mathsf{W}}_k^{\text{r.v.}}$. An estimator, a random variable, is always indicated by an asterisk alone, its realization, the estimate, by an asterisk and prime. Following the notation of Reference [8], physical vectors are denoted by Times Roman letters, and their corresponding 3×1 representations by the corresponding sans-serif letters.

[6]Note, however, footnote 4.

application both for Earth-orbiting and interplanetary spacecraft. Of particular interest in the QUEST work has been the QUEST measurement model, which is

$$\hat{\mathbf{W}}_k^{\text{r.v.}} = \hat{\mathbf{W}}_k^{\text{true}} + \Delta\hat{\mathbf{W}}_k^{\text{r.v.}}, \qquad \hat{\mathbf{W}}_k^{\text{true}} = A^{\text{true}}\hat{\mathbf{V}}_k, \qquad k = 1, \ldots, N \tag{3ab}$$

The measurement errors are zero-mean and Gaussian

$$\Delta\hat{\mathbf{W}}_k^{\text{r.v.}} \sim \mathcal{N}(\mathbf{0}, R_k), \qquad k = 1, \ldots, N \tag{3c}$$

and have a circle of error about the true value of the observations of radius σ_k, $k = 1, \ldots, N$.

$$R_k = \sigma_k^2 \left(I_{3\times3} - \hat{\mathbf{W}}^{\text{true}} \hat{\mathbf{W}}_k^{\text{true}\,T} \right), \qquad k = 1, \ldots, N \tag{3d}$$

The variances σ_k^2, $k = 1, \ldots, N$, are the sole parameters of the model. The individual direction measurements are assumed to be statistically independent. The circle of error in the tangent plane is less general than the more realistic ellipse of error, but leads to great simplification in the analytical results. This approximation is generally adequate for focal-plane sensors with limited fields of view (such as a star tracker) but has been employed also for many sensors for which the truthfulness of its representation of sensor errors may be justifiably questioned. Nonetheless, it has led to the development of many practical attitude estimators. The assumption that $\Delta\hat{\mathbf{W}}_k^{\text{r.v.}}$ is zero-mean and Gaussian also cannot be exactly correct [10], but is true to good approximation. Deviations in the attitude estimate due to this approximation are generally on the order of σ_k^2, which for $\sigma_k = 3$ arcsec, leads to an equivalent error in angle on the order of 5×10^{-5} arcsec, surely a negligible error. The reference vectors $\hat{\mathbf{V}}_k$, $k = 1, \ldots, N$ are assumed to be noise-free.

The Davenport q-algorithm [1, 9] constructs the optimal attitude estimate $A^{*\prime}$ by first constructing the attitude profile matrix B, defined as

$$B \equiv \sum_{k=1}^{N} a_k \hat{\mathbf{W}}_k' \hat{\mathbf{V}}_k^T \tag{4}$$

whence,

$$L(A) = \sum_{k=1}^{N} a_k - \text{tr}[B^T A] \equiv \lambda_\text{o} - g_A(A) \tag{5}$$

with $g_A(A)$ the *gain* function. From equation (5), the following quantities are defined

$$s \equiv \text{tr}B \tag{6a}$$

$$S \equiv B + B' \tag{6b}$$

$$\mathbf{Z} \equiv \left[B_{23} - B_{32},\, B_{31} - B_{13},\, B_{12} - B_{21} \right]^T \tag{6c}$$

and the 4×4 matrix K

$$K = \begin{bmatrix} S - sI_{3\times3} & \mathbf{Z} \\ \mathbf{Z}^T & s \end{bmatrix} \tag{7}$$

In terms of the quaternion [8] and Davenport's matrix K, the gain function may be written as

$$g_{\bar{q}}(\bar{q}) \equiv g_A(A(\bar{q})) = \bar{q}^T K \bar{q} \tag{8}$$

which is a maximum (and the loss function a minimum) for $\bar{q} = \bar{q}^{*\prime}$ with

$$K\bar{q}^{*\prime} = \lambda_{\max}\bar{q}^{*\prime} \tag{9}$$

and $\lambda_{\max}$ is the largest eigenvalue of K.[7] This is Davenport's q-algorithm. The earliest implementation of Davenport's q-algorithm was in SNAPLS (for SNAPshot Least-Squares), the attitude determination software system for the HEAO [11] mission (launched 1977, 1978 and 1979). The SNAPLS algorithm solved for $\bar{q}^{*\prime}$ by implementing Householder's method [12] to determine the four eigenvalues and eigenvectors of K.

The QUEST algorithm [1], whose details need not concern us in the present work, offered a very fast method for computing $\lambda_{\max}$ and the associated quaternion, the QUEST measurement model (*see above*), the method of sequential rotations, a more useful definition of the attitude error and of the attitude covariance matrix, a compact and easily calculable expression for the attitude estimate-error covariance matrix based on the QUEST measurement model, and a very fast data validation algorithm using a variable called TASTE (hence the expression "TASTE Test"). In addition, it was also shown in Reference [1] that the cost function would be minimized for constant λ_o if one chose

$$a_k = c/\sigma_k^2, \qquad k = 1, \ldots, N \tag{10}$$

In this case, it was later shown [10] that the Wahba attitude estimate was also the maximum-likelihood estimate of the attitude given the QUEST measurement model of equations (3). Furthermore, if one chose $c = 1$, then the loss function of equation (1) became the data-dependent part of the negative-log-likelihood function [13] of the attitude given the QUEST measurement model. This gave a firm statistical basis for the Wahba problem, rather than its being purely a mathematical curiosity.

The variable TASTE, which is central to the present study, is defined as

$$\text{TASTE} \equiv 2(\lambda_o - \lambda_{\max}) = 2L(A^*) \tag{11}$$

with the weights a_k being given by $1/\sigma_k^2$, that is, $c = 1$ in equation (10). Reference [1] chose c so that $\lambda_o = 1$.

THE STATISTICS OF TASTE

Given the above choice for the weights, we write for arbitrary A

$$L(A) = \frac{1}{2}\sum_{k=1}^{N}\frac{1}{\sigma_k^2}|\hat{\mathbf{W}}_k' - A\hat{\mathbf{V}}_k|^2 \tag{12}$$

[7] To be more exact, we should write K' and $\lambda_{\max}'$, since they are both functions of the realizations of random measurements.

The true value of the attitude, A^{true}, minimizes

$$L^{\text{true}}(A) \equiv \frac{1}{2} \sum_{k=1}^{N} \frac{1}{\sigma_k^2} |\hat{\mathbf{W}}_k^{\text{true}} - A\hat{\mathbf{V}}_k|^2 \tag{13}$$

and

$$\lambda_{\max}^{\text{true}} = \lambda_{\text{o}} = \frac{1}{\sigma_{\text{tot}}^2} \equiv \sum_{k=1}^{N} \frac{1}{\sigma_k^2} \tag{14}$$

We define now the attitude increment vector $\boldsymbol{\xi}$ [8] according to

$$A = A(\boldsymbol{\xi}) \equiv \delta A(\boldsymbol{\xi}) A^{\text{true}} \tag{15}$$

with

$$\begin{aligned} \delta A(\boldsymbol{\xi}) &\equiv \exp\{ [[\boldsymbol{\xi}]] \} \\ &= I_{3\times 3} + \frac{\sin |\boldsymbol{\xi}|}{|\boldsymbol{\xi}|} [[\boldsymbol{\xi}]] + \frac{1 - \cos |\boldsymbol{\xi}|}{|\boldsymbol{\xi}|^2} [[\boldsymbol{\xi}]]^2 \\ &= I_{3\times 3} + [[\boldsymbol{\xi}]] + O(|\boldsymbol{\xi}|^2) \end{aligned} \tag{16}$$

and [8]

$$[[\mathbf{u}]] \equiv \begin{bmatrix} 0 & u_3 & -u_2 \\ -u_3 & 0 & u_1 \\ u_2 & -u_1 & 0 \end{bmatrix} \tag{17}$$

$I_{3\times 3}$ is the 3×3 identity matrix. The matrix $\delta A(\boldsymbol{\xi})$ defined by equation (16) is exactly proper orthogonal. We anticipate that $\boldsymbol{\xi}^{*\prime}$ will be on the order of σ_{tot} with very large probability. The attitude increment vector is clearly a rotation vector [8]. Substituting now equation (15) into equation (12), we obtain after some manipulation

$$L_{\boldsymbol{\xi}}(\boldsymbol{\xi}) \equiv L(A(\boldsymbol{\xi})) = \frac{1}{2} \sum_{k=1}^{N} \frac{1}{\sigma_k^2} \left| \Delta \hat{\mathbf{W}}_k' + [[\hat{\mathbf{W}}_k^{\text{true}}]] \boldsymbol{\xi} \right|^2 + O(|\boldsymbol{\xi}|^3) \tag{18}$$

Let $S(\hat{\mathbf{W}}_k^{\text{true}})$ be any constant proper orthogonal matrix which accomplishes the transformation

$$S(\hat{\mathbf{W}}_k^{\text{true}}) \hat{\mathbf{W}}_k^{\text{true}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \equiv \hat{\mathbf{3}}, \qquad k = 1, \ldots, N \tag{19}$$

whence,

$$S(\hat{\mathbf{W}}_k^{\text{true}}) \, R_k \, S^T(\hat{\mathbf{W}}_k^{\text{true}}) = \sigma_k^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad k = 1, \ldots, N \tag{20}$$

Defining

$$\hat{\mathbf{U}}'_k \equiv \frac{1}{\sigma_k} S(\hat{\mathbf{W}}_k^{\text{true}})\hat{\mathbf{W}}'_k\,, \qquad k = 1, \ldots, N \tag{21a}$$

$$\hat{\mathbf{U}}_k^{\text{true}} \equiv \frac{1}{\sigma_k} S(\hat{\mathbf{W}}_k^{\text{true}})\hat{\mathbf{W}}_k^{\text{true}} = \frac{1}{\sigma_k}\hat{\mathbf{3}}\,, \qquad k = 1, \ldots, N \tag{21b}$$

we can write, keeping only terms to second order,

$$2L_{\xi}(\boldsymbol{\xi}) = \sum_{k=1}^{N} \left| \Delta\hat{\mathbf{U}}'_k + [[\,\hat{\mathbf{U}}_k^{\text{true}}\,]]\, S(\hat{\mathbf{W}}_k^{\text{true}})\,\boldsymbol{\xi} \right|^2 \tag{22}$$

Defining further

$$\Delta u'_{2k-1} \equiv \hat{\mathbf{1}}^T \Delta\hat{\mathbf{U}}'_k \quad \text{and} \quad \Delta u'_{2k} \equiv \hat{\mathbf{2}}^T \Delta\hat{\mathbf{U}}'_k\,, \qquad k = 1, \ldots, N \tag{23ab}$$

with

$$\hat{\mathbf{1}} \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \text{and} \qquad \hat{\mathbf{2}} \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{24ab}$$

the projections of $\hat{\mathbf{U}}'_k$ along the two axes perpendicular to $\hat{\mathbf{3}}$, respectively, and similarly,

$$H_{2k-1} \;\equiv\; -\hat{\mathbf{1}}^T [[\,\hat{\mathbf{U}}_k^{\text{true}}\,]]\, S(\hat{\mathbf{W}}_k^{\text{true}})\,, \qquad k = 1, \ldots, N \tag{25a}$$

$$H_{2k} \;\equiv\; -\hat{\mathbf{2}}^T [[\,\hat{\mathbf{U}}_k^{\text{true}}\,]]\, S(\hat{\mathbf{W}}_k^{\text{true}})\,, \qquad k = 1, \ldots, N \tag{25b}$$

We may write the loss function as

$$2L_{\xi}(\boldsymbol{\xi}) = \sum_{k=1}^{2N} |\Delta u'_k - H_k\,\boldsymbol{\xi}|^2 \tag{26}$$

The third component of $\hat{\mathbf{U}}'_k$ does not contribute to the loss function, since, in our measurement model, it is identically zero, and its sensitivity to $\boldsymbol{\xi}$ also vanishes by explicit construction. Note in particular that the $2N$ effective scalar measurement noise terms are independent and satisfy

$$\Delta u_k^{\text{r.v.}} \sim \mathcal{N}(0, 1)\,, \qquad k = 1, \ldots, 2N \tag{27}$$

The maximum-likelihood estimate of $\boldsymbol{\xi}$ is trivially

$$\boldsymbol{\xi}^{*\prime} = P \sum_{k=1}^{2N} H_k^T \Delta u'_k \tag{28a}$$

with

$$P^{-1} = \sum_{k=1}^{2N} H_k^T H_k \tag{28b}$$

and

$$\boldsymbol{\xi}^* \sim \mathcal{N}(\mathbf{0},\ P) \tag{29}$$

Writing

$$2L_{\xi}(\boldsymbol{\xi}) = \sum_{k=1}^{2N} |(\Delta u'_k - H_k\,\boldsymbol{\xi}^{*\prime}) - H_k\,(\boldsymbol{\xi} - \boldsymbol{\xi}^{*\prime})|^2 \tag{30}$$

we obtain straightforwardly

$$2L_{\xi}(\boldsymbol{\xi}^{\text{true}}) = 2L(\boldsymbol{\xi}^{*\prime}) + (\boldsymbol{\xi}^{*\prime} - \boldsymbol{\xi}^{\text{true}})^T P^{-1} (\boldsymbol{\xi}^{*\prime} - \boldsymbol{\xi}^{\text{true}}) \tag{31}$$

The absence of a cross term is a natural consequence of the Luenberger projection theorem [14], but we obtain it equally well in this trivial example by direct calculation. By definition,

$$\boldsymbol{\xi}^{\text{true}} = \mathbf{0} \tag{32}$$

and from equation (26),

$$2L^{\text{r.v.}}(A^{\text{true}}) = \sum_{k=1}^{2N} |\Delta u_k^{\text{r.v.}}|^2 \sim \chi^2(2N) \tag{33}$$

a χ^2 random variable with $2N$ degrees of freedom. Clearly, from equation (29), one has that

$$(\boldsymbol{\xi}^* - \boldsymbol{\xi}^{\text{true}})^T P^{-1} (\boldsymbol{\xi}^* - \boldsymbol{\xi}^{\text{true}}) \sim \chi^2(3) \tag{34}$$

Thus,

$$\chi^2(2N) = \text{TASTE} + \chi^2(3) \tag{35}$$

Since the two terms in the right member of equation (35) are statistically independent, then by Cochran's theorem or, equivalently, Fisher's theorem (both Reference [15]), equation (35) can be true only if[8]

$$2L_{\xi}(\boldsymbol{\xi}^*) \equiv \text{TASTE} \sim \chi^2(2N - 3) \tag{36}$$

As an immediate consequence of equation (36), it follows that

$$E\{\,\text{TASTE}\,\} = 2N - 3 \qquad \text{and} \qquad \text{Var}\{\,\text{TASTE}\,\} = 2(2N - 3) \tag{37}$$

for the expectation and the variance of TASTE.

THE IMPLEMENTATION OF THE TASTE TEST IN STAR TRACKERS

Unlike the Ball CT-401 star tracker used on Magsat thirty years ago, a modern CCD star tracker can track from five to fifty stars simultaneously. Star identification is carried out by pattern recognition, i.e., by comparing the angular separations of stars observed by the star tracker with those in a star catalogue.

[8]In the author's earliest comment in the Magsat ground support software in 1979, he stated only that TASTE was approximately $\chi^2(2N)$.

Let us assume that star 1 has been misidentified and has a "position" error ϵ, which we will assume to be along the star-tracker x axis.

$$\hat{\mathbf{U}}_1' \to \hat{\mathbf{U}}_1' + \begin{bmatrix} \epsilon/\sigma \\ 0 \\ 0 \end{bmatrix} \tag{38}$$

where σ is the single-star accuracy of the star-tracker. The loss function is now given by

$$2L_{\boldsymbol{\xi}}(\boldsymbol{\xi}) = |\Delta u_1' + \epsilon/\sigma - H_1\,\boldsymbol{\xi}|^2 + \sum_{k=2}^{2N} |\Delta u_k' - H_k\,\boldsymbol{\xi}|^2 \tag{39}$$

Here $\Delta u_1'$ is the value which would have occurred for a correctly identified star. Differentiating with respect to $\boldsymbol{\xi}$ yields

$$\begin{aligned} \frac{\partial L_{\boldsymbol{\xi}}(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}} &= -(\epsilon/\sigma)\,H_1^T - \sum_{k=1}^{2N} H_k^T[\Delta u_k' - H_k\,\boldsymbol{\xi}] \\ &= -(\epsilon/\sigma)\,H_1^T - P^{-1}(\boldsymbol{\xi}^{*\prime} - \boldsymbol{\xi}) \end{aligned} \tag{40}$$

Optimizing $L_{\boldsymbol{\xi}}(\boldsymbol{\xi})$ now leads to an incorrect optimal estimate $\boldsymbol{\xi}^{**\prime}$ given by

$$\boldsymbol{\xi}^{**\prime} = \boldsymbol{\xi}^{*\prime} + (\epsilon/\sigma)\,PH_1^T \tag{41}$$

with $\boldsymbol{\xi}^{*\prime}$ and P given by equations (28).

For a star tracker, the measurements are clustered about the star-tracker boresight, so that we can make the approximation

$$S(\hat{\mathbf{W}}_k^{\text{true}}) \approx I_{3\times 3} \tag{42}$$

Then,

$$H_1^T \approx -\frac{1}{\sigma}\,\hat{\mathbf{2}} \tag{43}$$

and

$$P \approx \frac{\sigma^2}{N} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \text{GDOP}^2 \end{bmatrix} \tag{44}$$

and GDOP is the geometric distortion of precision on the order of 17 for a star-tracker with a typical 8 deg by 8 deg field of view. (If equation (42) is true exactly, then GDOP is infinite.) Substituting equations (43) and (44) into equation (41) leads to

$$\boldsymbol{\xi}^{**\prime} \approx \boldsymbol{\xi}^{*\prime} - (\epsilon/N)\,\hat{\mathbf{2}} \tag{45}$$

where $\boldsymbol{\xi}^{**\prime}$ denotes the value of $\boldsymbol{\xi}$ which minimizes the loss function of equation (39), and $\boldsymbol{\xi}^{*\prime}$ denotes the value of $\boldsymbol{\xi}$ which minimizes the loss function of equation (22). The effect of the misidentified star is to alter the estimate of $\boldsymbol{\xi}$ by ϵ/N about one axis normal to the star-tracker boresight. The factor $1/N$ in equation (45) shows that the optimal attitude estimate effectively spreads the error over all N star observations.

If we insert equations (43) and (45) into equation (39), we obtain straightforwardly but after much labor

$$2L_{\xi}(\boldsymbol{\xi}^{**\prime}) = 2L_{\xi}(\boldsymbol{\xi}^{*\prime}) - \frac{2\epsilon}{\sigma} X + \left(1 - \frac{1}{N}\right) \left(\frac{\epsilon}{\sigma}\right)^2 \tag{46}$$

and

$$\begin{aligned} X &= (\Delta u_1' - H_1 \boldsymbol{\xi}^{*\prime}) + \frac{1}{N} \sum_{\substack{k=1 \\ k \text{ odd}}}^{2N} (\Delta u_k' - H_k \boldsymbol{\xi}^{*\prime}) \\ &\equiv X_1 + X_2 \end{aligned} \tag{47}$$

The random variable X_1 has mean zero and variance unity. The random variable X_2 has mean zero and variance $(2N-3)/N^2$. Thus, by the Cauchy-Schwartz inequality, we have

$$1 \le \mathrm{Var}\{ X \} \le \left[1 + \sqrt{\frac{2N-3}{N^2}} \right]^2 \le \frac{4 + 2\sqrt{3}}{3} \approx 2.488 \tag{48}$$

The maximum value of the central member of equation (48) occurs when $N = 3$. The limiting value 1 is achieved as $N \to \infty$. Note that X and $2L_{\xi}(\boldsymbol{\xi}^{*\prime})$ are correlated, making an exact calculation of the variance of $2L_{\xi}(\boldsymbol{\xi}^{**\prime})$ difficult. However, from equation (48) we see that even for $N = 5$, the standard deviation of X is relatively small compared to the mean and standard deviation of of $2L_{\xi}(\boldsymbol{\xi}^{*\prime})$, which are $2N - 3$ and $\sqrt{2(2N-3)}$, respectively.

To detect that a star must have been misidentified, we must have that (ϵ/σ) be large compared to $2N - 3$. Neglecting X in equation (46) in this case and substituting equation (36), we have approximately

$$2L_{\xi}(\boldsymbol{\xi}^{**\prime}) \approx 2N - 3 \pm \sqrt{2(2N-3)} + \left(1 - \frac{1}{N}\right) (\epsilon/\sigma)^2 \tag{49}$$

We might call $2L_{\xi}(\boldsymbol{\xi}^{**\prime})$ the "bad TASTE."

If we consider an example with numbers characteristic of the Magsat mission[9] with $N = 3$, $\sigma = 13$ arcsec, and $\epsilon = 0.5$ deg, then very approximately,

$$2L_{\xi}(\boldsymbol{\xi}^{**\prime}) \approx 3 \pm 2.45 + 13,000 \tag{50}$$

which is a 5000-sigma event. "Sigma" here means the standard deviation of TASTE without the position error, not the single-star accuracy of the star tracker.

For the Lockheed-Martin AST-201 star tracker of the WMAP mission [16], which typically observed around 25 stars and had a single-star accuracy of 10 arcsec we have for the same position error of the misidentified star

$$2L_{\xi}(\boldsymbol{\xi}^{**\prime}) \approx 50 \pm 10 + 32,000 \tag{51}$$

[9]The observed directions in the Magsat mission were not confined to a narrow cone, so the example does not really apply, although the order of magnitudes should be comparable.

which is an approximately 3200-sigma event.

Star identification brings an additional complication. Star identification is usually accomplished by comparing the angular separation of stars observed in the field of view of the star tracker with those in a star catalogue. Generally, star searches in the catalogue are done within a cone of approximately 4 arcmin full width. Thus, in this case, ϵ is typically no more than 2 arcmin. For the WMAP star tracker we have typically, for this value of ϵ,

$$2L_\xi(\boldsymbol{\xi}^{**\prime}) \approx 50 \pm 10 + 144 \tag{52}$$

and a misidentified star with a position error of this magnitude results in a 10-sigma event. If star identification were performed with a tolerance of of 1 arcsec (full width) for all $N(N-1)/2$ observed star pairs, then the TASTE would show no more than one-sigma anomalies and not be very useful. It is not, however, the general practice to match all star pairs in star identification. The MSX spacecraft chose the minimum magnitude of its catalogue stars to be such that the on-board star catalogue would contain about one star per square degree (about 65,000 stars).

For the moment, the TASTE test can still be useful for modern CCD star trackers.

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