M. Shuster

Nuclear Physics B87 (1975) 109--126

THE ABC EFFECT IN THE REACTION NN → dππ

I. BAR-NIR *
Division T.C., CERN, Geneva, Switzerland

T. RISSER
Department of Physics, University of California, Santa Barbara, California, USA

M.D. SHUSTER
Department of Physics and Astronomy, Tel-Aviv University, Ramat Aviv, Tel-Aviv, Isreal

Received 31 May 1974

A simple peripheral model for the reaction NN → NNππ is extended to the reaction NN → dππ. Predictions for the differential and total cross sections are compared with recent experiments. The enhancement of the deuteron recoil-momentum spectrum near the ππ threshold (ABC effect) is well reproduced.

1. Introduction

In a previous paper [1] arguments were presented to demonstrate that the ABC effect as observed in the reaction

$$N + N \rightarrow d + \pi + \pi$$

was only the manifestation of the excitation of two Δ(1236) resonances in the two-baryon system. The once supposed ππ resonance observed at a missing mass about 300 MeV was shown to reflect only the maximum overlap of the Breit-Wigner functions for the decays of the two Δ's and the virtual decay of the deuteron into two nucleons.

The model presented in that paper reproduced qualitatively all the peculiarities of the reaction at intermediate energies. These included:

(i) the enhancement of the cross section (relative to the dππ phase space) for the reaction np → dππ at total c.m. energies around $2M_\Delta c^2$ for small effective ππ masses (ABC effect);

(ii) the absence of a similar phenomenon in the reaction pp → dππ;

* On leave from Department of Physics and Astronomy, Tel-Aviv University, Ramat Aviv, Tel-Aviv, Israel.

(iii) the prediction of a broad enhancement in the first reaction for the largest permitted $\pi\pi$ effective masses, which had not then been observed.

However, this qualitative demonstration made a number of simplifications: a) the πN interaction was replaced by an equivalent s-wave interaction which reproduced the total cross section; b) the deuteron binding energy was taken to vanish; and c) no account was taken of the nucleon or deuteron spins. For these reasons no attempt was made in the earlier work to make a quantitative comparison between theory and experiment. In particular, there was no attempt made to calculate absolute cross sections for this reaction. It is the purpose of the present report to remove these deficiencies.

A realistic peripheral model for the reaction NN → NN$\pi\pi$ has been adapted to the present study. The calculation for the ABC effect is thus performed in a manner consisted with calculations of this reaction. The model, therefore, also contains no adjustable parameters.

Since the nucleons and deuteron in this reaction are relativistic particles, it will be necessary to treat these in a Lorentz-covariant fashion. On the other hand, all practical theories of the deuteron are non-relativistic and we will need, therefore, to accommodate the Schrödinger wave function of the deuteron to our relativistic description. Numerous prescriptions [2] exist for the case where only one nucleon of the NNd vertex is off the mass shell. The case where both nucleons are not on the mass shell has not received any practical attention to our knowledge. In the present work, we have chosen the simplest possible prescription, whose only justification is that it leads to the non-relativistic expression in that limit.

The spin and isospin summations have been performed exactly. Expressions for these whose application is more general than the present work have been derived in appendices A and B.

2. The model

2.1. The OPE amplitude

As in the previous work we restrict our attention to the mechanism of one-pion exchange as shown in the Feynman graph of fig. 1. Putting aside for the moment the requirement that the amplitude must be antisymmetric in the nucleon variables and symmetric in the pion variables the amplitude of fig. 1 must have the following form

$$\mathcal{M}(p_1s_1, p_2s_2, k_1k_2, p_d, S) = \int d^4n \frac{i}{\kappa^2 - m_\pi^2} \epsilon^*_\mu(S) \overline{\Gamma}^\mu_{\alpha\beta}(d+n, d-n)$$

$$\times {}_\alpha\left(\frac{1}{\gamma\cdot(d+n) - M} \mathcal{F}(\kappa, p_1; k_1, d+n) u(p_1, s_1)\right)$$

$$\times {}_\beta\left(\frac{1}{\gamma\cdot(d-n) - M} \mathcal{F}(-\kappa, p_2; k_2, d-n) u(p_2, s_2)\right) . \qquad (1)$$

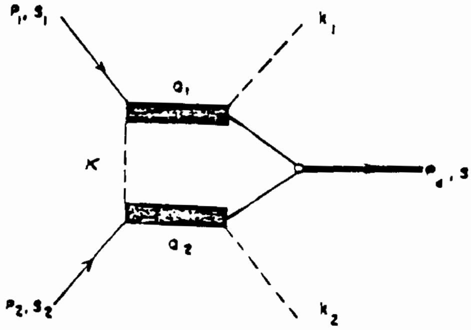

Fig. 1. OPE model for NN → dππ.

Here M is the nucleon mass, $d = \frac{1}{2}p_d$, and $\kappa = d+n+k_1-p_1$ is the momentum of the exchanged pion. $\mathcal{F}(k, p; k', p')$ is the phenomenological vertex for πN scattering, $\epsilon^*_\mu(S)$ is the complex conjugate of the deuteron polarisation vector, and $\Gamma^\mu(P_a, P_b)$ is the deuteron vertex function, a bilinear operator in the spin space of the two nucleons. (In eq. (1) we have written the spin indices explicitly.) The adjoint is given by

$$\overline{\Gamma^\mu}(P_a, P_b) = \Gamma^{\mu\dagger}(P_a, P_b)\gamma_0^{(a)}\gamma_0^{(b)}\,, \tag{2}$$

S is the projection of the deuteron spin.

Unless otherwise specified the letters κ, d, n, p, k, Q, and K denote four-vectors. Three-vectors will be denoted by bold face. The letter q will always denote a Lorentz scalar. Throughout we use the conventions of Bjorken and Drell [3]. We do not write the isospin matrix elements explicitly. These are calculated in appendix A.

2.2. The πN → πN vertex

If we assume that the pion-nucleon scattering proceeds entirely through the intermediate $\Delta(\frac{3}{2}, \frac{3}{2}^+)$ resonance, then apart from the isospin matrix element the phenomenological πN → πN vertex, when all four particles are on the mass shell, has the form

$$T_{\pi N \to \pi N} = \bar{u}(p', s')\,\mathcal{F}(p, k; p', k')\,u(p, s)\,,$$

$$\mathcal{F}(p, k, p', k') = G\,\frac{M_\Delta \Gamma_\Delta(q)/q^3}{Q^2 - M_\Delta^2 - iM_\Delta \Gamma_\Delta(q)}\,(k'_\mu U_{\mu\nu}(Q) k_\nu)\,, \tag{3}$$

where the primed (unprimed) quantities refer to outgoing (incoming) particles. k denotes a pion momentum and p a nucleon momentum. Q is the total four-momentum of the πN system,

$$Q = k + p = k' + p' \tag{4}$$

and q, a Lorentz scalar, is the (on-shell) decay momentum of the Δ

$$q = \sqrt{\frac{1}{4Q^2}\,\lambda(Q^2, M^2, m_\pi^2)}\,, \tag{5}$$

with

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz\,,$$

$U_{\mu\nu}(Q)$ is the projection operator for a spin-$\frac{3}{2}$ particles, which we take to be

$$U_{\mu\nu}(Q) = -\frac{\gamma\cdot Q + M_\Delta}{6M_\Delta}\left[3g_{\mu\nu} - \gamma_\mu\gamma_\nu - \frac{1}{M_\Delta}(\gamma_\mu Q_\nu - \gamma_\nu Q_\mu) - \frac{2}{M_\Delta^2}Q_\mu Q_\nu\right], \tag{6}$$

M_Δ and $\Gamma_\Delta(q)$ being the mass (1236 MeV) and the phenomenological width of the Δ, respectively. In the formalism of Benecke and Dürr [4] the phenomenological width takes the form

$$\Gamma_\Delta(q) = \gamma\frac{M_\Delta q}{M q_r}\,\frac{u_1(R_\Delta q)}{u_1(R_\Delta q_r)}\,, \tag{7}$$

where q_r is the (on-shell) decay momentum at resonance

$$q_r = \sqrt{\frac{1}{4M_\Delta^2}\,\lambda(M_\Delta^2, M^2, m_\pi^2)} \tag{8}$$

and $u_1(x)$ is the function

$$u_1(x) = \frac{1}{2x^2}\left[\frac{2x^2+1}{4x^2}\ln(4x^2+1) - 1\right]. \tag{9}$$

Independently of the values taken by the other parameters the phenomenological amplitude will attain the unitary limit at resonance provided G is chosen as

$$G = 48\pi\frac{M_\Delta^2}{(M_\Delta+M)^2 - m_\pi^2}\,,$$

$$\approx 50\,, \tag{10}$$

which can be obtained most simply by calculating the forward scattering amplitude at resonance using the prescription of eq. (3) and applying the optical theorem.

For the case where one of the pions is not on the mass shell the amplitude must be modified by including a factor (essentially a vertex correction) for that pion

$$V(Q^2, M^2, \kappa^2) = \sqrt{(q/q_\kappa)^5 \Gamma_\Delta(q_\kappa)/\Gamma_\Delta(q)}\,, \tag{11}$$

with

$$q_\kappa = \sqrt{\frac{1}{4Q^2}\,\lambda(Q^2, M^2, \kappa^2)} \tag{12}$$

the "off-shell" decay momentum of the Δ, and κ the pion four-momentum.

This parametrization was investigated extensively by Wolf [5], who was able to reproduce a number of phenomena over a wide range of energies using the parameters

$$\gamma = 0.114\ \mathrm{GeV}\,, \qquad R_\Delta = 2.2\ (\mathrm{GeV}/c)^{-1}\,. \tag{13}$$

In particular, Wolf studied the reaction $NN \to NN\pi\pi$ at the same c.m. energy as the present study.

(It should be noted that we have chosen to write the amplitude for πN scattering in terms of the nucleon spinors (since this will simplify the spin sum) in contrast to Wolf, who wrote his amplitude explicitly as a function of the on- and off-shell momenta and angles (since he required only the total πN cross section). For this reason the amplitude of eq. (3) contains essentially an additional factor (q_κ/q) compared to the equivalent expression in Wolf. This has been compensated by including an additional factor (q/q_κ) in the vertex correction of eq. (11) above.)

2.3. The deuteron vertex

As stated earlier it is not the intent of this work to add to our present understanding of the relativistic deuteron vertex. We restrict ourselves, therefore, to the more modest problem of incorporating the Schrödinger wave function of the deuteron into the amplitude without destroying Lorentz covariance. To this end we note that if we make the identity

$$\frac{1}{\gamma^{(p)}\cdot p_p - M}\,\frac{1}{\gamma^{(n)}\cdot p_n - M}\,\Gamma^\mu(p_p, p_n)\epsilon_\mu(S) = (2\pi)^4\,\delta\left(\frac{p_d\cdot n}{M_d}\right)\psi_d(n, d, S)\,, \tag{14}$$

where $p_d = p_p + p_n = 2d$ is the total four-momentum and $n = \frac{1}{2}(p_p - p_n)$ is the relative four-momentum, then eq. (1) will reduce to the usual expression of non-relativistic quantum mechanics provided that $\psi_d(n, d, S)$ reduces in a suitable fashion to the Schrödinger wave function in the non-relativistic limit. The δ-function in eq. (14) arises from the fact that the Schrödinger wave function in coordinate space does not depend on the relative time.

The simplest prescription for $\psi_d(n, d, S)$ is

$$\psi_{\rm d}(n,d,S) = \phi(q_{\rm d}) \sum_{s_{\rm p},s_{\rm n}} R(n,d,s_{\rm p},s_{\rm n},S)u(d+n,s_{\rm p})u(d-n,s_{\rm n}) , \tag{15}$$

where $\phi(q_{\rm d})$ is the usual radial wave function in momentum space made invariant by writing it as a function of the deuteron "decay"-momentum

$$q_{\rm d} = \sqrt{-n^2} , \tag{16}$$

and the $R(n,d,s_{\rm p},s_{\rm n},S)$ are coefficients which couple the two spinors to a quantity of total-spin unity and with unit normalization. The exact form of these coefficients will not concern us. For the special case, $n = 0$, these must reduce to the familiar Clebsch-Gordan coefficients [6]

$$R(0,d,s_{\rm p},s_{\rm n},S) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ s_{\rm p} & s_{\rm n} & S \end{bmatrix} . \tag{17}$$

Except for the case $n = 0$, it is not obvious that the spinor coupling of eq. (15) can be made consistent with Lorentz covariance. It will turn out for our purposes that eq. (17) is sufficient to insure the Lorentz covariance of the amplitude.

The wave function of eq. (15) takes account only of the deuteron S-state. Since the largest contributions to the amplitude will arise from small values of $q_{\rm d}$, we expect the effect of the deuteron D-state to be very small. We have taken $\phi(q_{\rm d})$ to be the usual Hulthén wave function. In momentum space this is

$$\phi(q_{\rm d}) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{\pi^2(\alpha-\beta)^2}} \left(\frac{1}{\alpha^2+q_{\rm d}^2} - \frac{1}{\beta^2+q_{\rm d}^2}\right) , \tag{18}$$

with

$$\alpha = 45\ {\rm MeV}/c , \qquad \beta = 7\alpha .$$

2.4. *A useful approximation*

Collecting all the above expressions the amplitude becomes

$$\begin{aligned} &\mathcal{M}(p_1 s_1, p_2 s_2, k_1, k_2, p_{\rm d} S) \\ &\quad = -i \int {\rm d}^4 n (2\pi)^4\, \delta\left(\frac{p_{\rm d}\cdot n}{M_{\rm d}}\right) \phi(q_{\rm d}) \sum_{s_{\rm p},s_{\rm n}} R(n,d,s_{\rm p},s_{\rm n},S) \\ &\quad \times \frac{1}{\kappa^2-m_\pi^2}\, V(Q_1^2, M^2, \kappa^2)\, T(Q_1^2) (\bar{u}(d+n,s_{\rm p}) k_{1\mu} U_{\mu\nu}(Q_1) \kappa_\nu u(p_1,s_1)) \\ &\quad \times V(Q_2^2, M^2, \kappa^2)\, T(Q_2^2) (\bar{u}(d-n,s_{\rm n}) k_{2\rho} U_{\rho\sigma}(Q_2) \kappa_\sigma u(p_2,s_2)) , \end{aligned} \tag{19}$$

with

$$Q_1 = p_1 + \kappa \,, \tag{20a}$$

$$Q_2 = p_2 - \kappa \,, \tag{20b}$$

$$T(Q_i^2) = \frac{GM_\Delta \, \Gamma_\Delta(q_i)/q_i^3}{Q_i^2 - M_\Delta^2 - iM_\Delta \, \Gamma_\Delta(q_i)} \,, \qquad i = 1, 2 \,, \tag{21}$$

and q_i is the on-shell decay momentum defined in eq. (8).

We now make the approximation that the spinor matrix elements do not depend strongly on the momentum n and evaluate these at the point $n = 0$. Such an approximation is not unreasonable since the deuteron wave function $\phi(\sqrt{-n^2})$ decreases rapidly with increasing $\sqrt{-n^2}$. (Note that n is always space-like in the integral.) Thus

$$\mathcal{M}(p_1 s_1, p_2 s_2, k_1, k_2, p_d S) = J \sum_{s_p s_n} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ s_p & s_n & S \end{bmatrix}$$

$$\times (\bar{u}(d, s_p) k_{1\mu} U_{\mu\nu}(\bar{Q}_1) \bar{\kappa}_\nu u(p_1 s_1))(\bar{u}(d, s_n) k_{2\rho} U_{\rho\sigma}(\bar{Q}_2) \bar{\kappa}_\sigma u(p_2 s_2)) \,, \tag{22}$$

with

$$J = (2\pi)^4 \int d^4 n \, \delta\left(\frac{p_d \cdot n}{M_d}\right) \phi(q_d) \frac{-i}{\kappa^2 - m_\pi^2} \; V(Q_1^2, M^2, \kappa^2) \, T(Q_1^2)$$

$$\times \, V(Q_2^2, M^2, \kappa^2) \, T(Q_2^2) \,, \tag{23}$$

and

$$\bar{\kappa} = d + k_1 - p_1 \,, \tag{24a}$$

$$\bar{Q}_1 = p_1 + \bar{\kappa} \,, \tag{24b}$$

$$\bar{Q}_2 = p_2 - \bar{\kappa} \,, \tag{24c}$$

the equivalent quantities with $n = 0$. The amplitude given by eq. (22) is manifestly Lorentz covariant.

2.5. Spin and isospin

Turning our attention now to the requirements of symmetry, we note that the amplitude for our process must be invariant under simultaneous interchange of the pion space and nucleon space-spin variables. Hence, it is sufficient that the amplitude have the proper symmetry in one or the other of these variables. (This case is similar to that encountered for two-nucleon matrix elements of symmetric operators where it is sufficient to antisymmetrize either the initial or the final two-nucleon state.) Thus if the two nucleons are initially in a state of vanishing isospin it follows that

$$\mathcal{M}(I=0) = C_0[\mathcal{M}(p_1s_1, p_2s_2, k_1, k_2, p_d S) + \mathcal{M}(p_1s_1, p_2s_2, k_2, k_1, p_d S)], \tag{25a}$$

and for unit isospin

$$\mathcal{M}(I=1) = C_1[\mathcal{M}(p_1s_1, p_2s_2, k_1, k_2, p_d S) - \mathcal{M}(p_1s_1, p_2s_2, k_2, k_1, p_d S)], \tag{25b}$$

where C_0 and C_1 are the respective isospin matrix elements, which we have not written explicitly until now. These have the values (see appendix A)

$$C_0 = 2/\sqrt{3}, \qquad C_1 = \tfrac{5}{9}\sqrt{2}. \tag{26}$$

For the processes actually observed in the laboratory we have

$$\mathcal{M}(\mathrm{pp} \to \mathrm{d}\pi\pi) = C_1\mathcal{M}(k_1, k_2) - C_1\mathcal{M}(k_2, k_1), \tag{25c}$$

$$\mathcal{M}(\mathrm{np} \to \mathrm{d}\pi\pi) = -\frac{C_0 - C_1}{\sqrt{2}}\mathcal{M}(k_1, k_2) - \frac{C_0 + C_1}{\sqrt{2}}\mathcal{M}(k_2, k_1), \tag{25d}$$

where for simplicity we write only the pion momenta explicitly.

Thus, in general, for an arbitrary isospin channel we have

$$\mathcal{M} = \sum_{s_p s_n} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ s_p & s_n & S \end{bmatrix}$$
$$\times\, [\alpha_+(\bar{u}(d, s_p)A\, u(p_1, s_1))(\bar{u}(d, s_n)B\, u(p_2, s_2))$$
$$+\, \alpha_-(\bar{u}(d, s_p)A'\, u(p_1, s_1))(\bar{u}(d, s_n)B'\, u(p_2, s_2))], \tag{26}$$

where

$$\alpha_+ = C_+ J, \tag{27a}$$

$$\alpha_- = C_- J', \tag{27b}$$

with C_+ and C_- the coefficients of $\mathcal{M}(k_1, k_2)$ and $\mathcal{M}(k_2, k_1)$, respectively, in eq. (25) above. J is given by eq. (23), J' by the same expression with κ replaced by κ'

$$\kappa' = d + n + k_2 - p_1 \,, \tag{28}$$

$$A = k_{1\mu} U_{\mu\nu}(\bar{Q}_1)\bar{\kappa}_\nu \,, \tag{29a}$$

$$B = k_{2\mu} U_{\mu\nu}(\bar{Q}_2)\bar{\kappa}_\nu \,, \tag{29b}$$

$$A' = k_{2\mu} U_{\mu\nu}(\bar{Q}_2)\bar{\kappa}'_\nu \,, \tag{29c}$$

$$B' = k_{1\mu} U_{\mu\nu}(\bar{Q}_1)\bar{\kappa}'_\nu \,, \tag{29d}$$

with

$$\bar{\kappa}' = d + k_2 - p_1 \,. \tag{30}$$

In the same notation the complex conjugate of the amplitude is

$$\mathcal{M}^* = \sum_{s_p s_n} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ s_p & s_n & S \end{bmatrix}$$

$$\times \{\alpha_+^* (\bar{u}(p_1, s_1)\tilde{A}\; u(d, s_p))(\bar{u}(p_2, s_2)\tilde{B}\; u(d, s_n))$$

$$+ \alpha_-^* (\bar{u}(p_1, s_1)\tilde{A}'\, u(d, s_p))(\bar{u}(p_2, s_2)\tilde{B}'\, u(d, s_n))\} \,, \tag{31}$$

where

$$\tilde{X} = \gamma_0 X^\dagger \gamma_0 \,. \tag{32}$$

The summation over deuteron spins and averaging over initial nucleon spins (see appendix B) leads to

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{2} \frac{1}{(2M)^4}$$

$$\times \{|\alpha_+|^2 \,[\mathrm{Tr}((\not{d}+M)A(\not{p}_1+M)\tilde{A}(\not{d}+M)B(\not{p}_2+M)\tilde{B})$$

$$+ \mathrm{Tr}((\not{d}+M)A(\not{p}_1+M)\tilde{A})\mathrm{Tr}((\not{d}+M)B(\not{p}_2+M)\tilde{B})]$$

$$+ \alpha_+^* \alpha_- [A \to A', B \to B'] + \alpha_-^* \alpha_+ [\tilde{A} \to \tilde{A}', \tilde{B} \to \tilde{B}']$$

$$+ |\alpha_-|^2 [A \to A', B \to B', \tilde{A} \to \tilde{A}', \tilde{B} \to \tilde{B}']\} \,. \tag{33}$$

The second and third lines are, in fact, complex conjugates. The brackets of the last three terms denote the substitutions which must be made in the first term to reach the necessary expression.

2.6. Kinematics and phase space

The differential cross section for the process NN → dππ is given by [3]

$$d\sigma = F \frac{d^3k_1}{2\omega_1(2\pi)^3} \frac{d^3k_2}{2\omega_2(2\pi)^3} \frac{M_d}{E_d} \frac{d^3p_d}{(2\pi)^3}$$

$$\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2 - p_d) \langle |\mathcal{M}|^2 \rangle , \tag{34}$$

where

$$F = M^2/\sqrt{(p_1 \cdot p_2)^2 - M^4} \tag{35}$$

is the invariant flux factor for the incident particles. The deuteron state has been normalized according to the prescription

$$\langle p'_d S' | p_d S \rangle = \frac{E_d}{M_d} (2\pi)^3 \delta^{(3)}(p_d - p'_d) \delta_{SS'} . \tag{36}$$

Eliminating unobserved variables the expression for the differential cross sections becomes

$$\left(\frac{d^2\sigma}{d\Omega_d d|p_d|} \right)_{lab} = \frac{1}{(2\pi)^5} \left(\frac{M M_d |p_d|^2}{8|p_1| E_d} \right)_{lab} \sqrt{\frac{K^2 - 4m_\pi^2}{K^2}} \left(\int d\Omega_k \langle |\mathcal{M}|^2 \rangle \right)_{K\ c.m.} , \tag{37}$$

with

$$K = k_1 + k_2 , \qquad k = \tfrac{1}{2}(k_1 - k_2) . \tag{38}$$

The subscripts "lab" and "K c.m." denote that the expressions so labelled are to be evaluated in the lab frame or in the c.m. frame of the two pions ($K = 0$). The quantities preceding the integral in eq. (37) are naturally the kinematic phase space.

3. Numerical results and discussion

The differential cross sections for the reactions

$$p + p \to d + (\pi\pi)^+ , \qquad n + p \to d + (\pi\pi)^0 ,$$

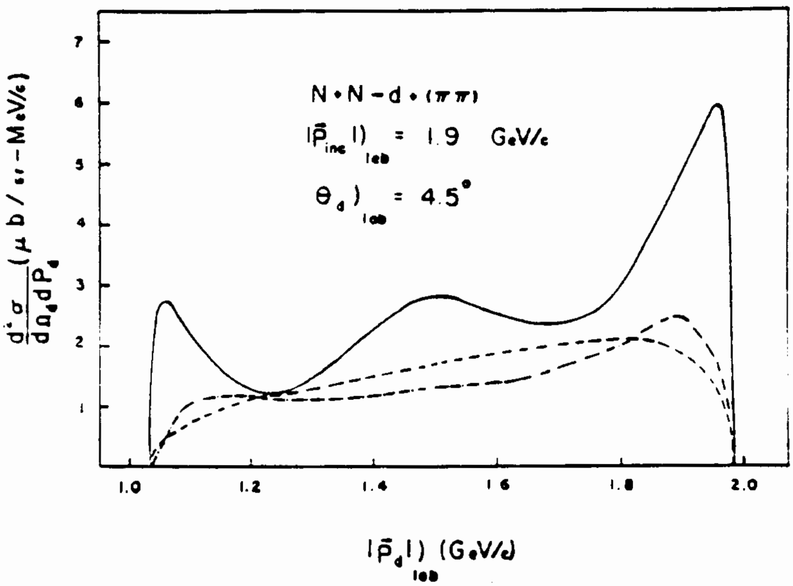

Fig. 2. Predicted differential cross sections for the reactions np → dππ (solid line) and pp → dππ (dotted and dashed line) and the dππ phase space (dashed line).

as given by eq. (37) and the preceding were calculated using Monte-Carlo methods on the CERN CDC 7600 computer. The results are shown in fig. 2.

As in the previous calculation the cross section for the reaction np → dππ shows pronounced enhancements at missing masses close to $2m_\pi$ (corresponding to the lowest and highest possible values of the deuteron recoil momentum in the laboratory) and at the largest possible missing masses (the central peak). As was shown in ref. [1] it is for these values of the deuteron recoil momentum that the two Δ's can be simultaneously at resonance for all values of Ω_k. The suppression of an enhancement in the isospin-one channel follows again from the large cancellations in this amplitude as evidenced by eq. (25b). We remark that the cross section for the isospin-one channel compared to that for the isospin-zero channel is relatively much larger in the more realistic model presented here than it was in our earlier more primitive model of ref. [1]. This, as well as the greater sharpness of the ABC peaks, results from our now proper treatment of the particle spins.

In fig. 3 the calculation for the reaction np → dππ is compared with recent data of the Saclay Deuteron Group [7] for the reaction

n + p → d + missing mass

at an incident neutron (lab) momentum of 1.88 GeV/c. The analysis of the Saclay data is still preliminary. The final results may change perhaps by 10 or 20% [8].

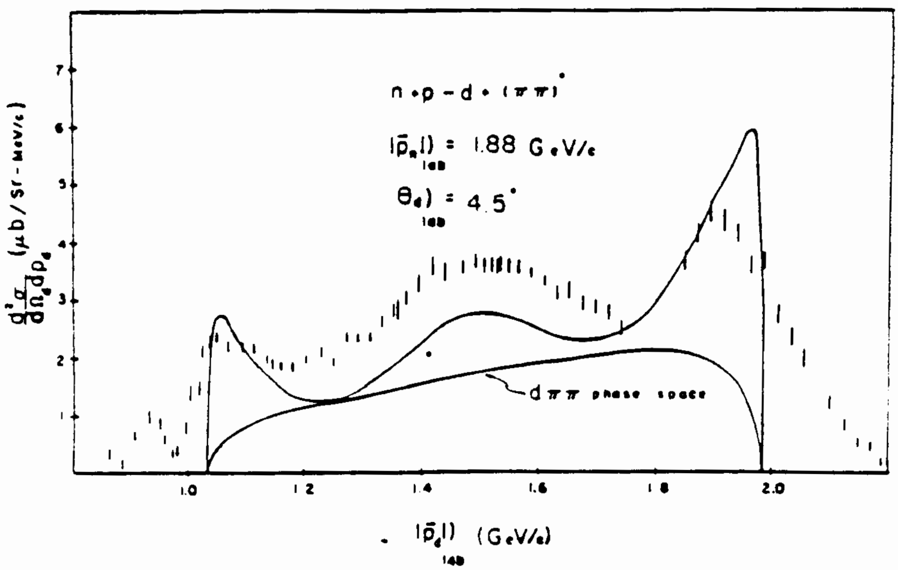

Fig. 3. Comparison of the predicted differential cross section for the reaction np → dππ with the Saclay data. The one-pion peak is also observed.

We remark that the intervals in the deuteron recoil-momentum spectrum between the end points of the dππ phase space and the positions of the poles for single-pion production are forbidden kinematically. Thus the "tails" of the observed cross sections in those regions must reflect the experimental resolution. Since the area under each of these "tails" is equal roughly to the "missing" area under the corresponding ABC peaks, we may conclude that folding an experimental resolution into our calculation would considerably improve the agreement with experiment at the ABC peaks. The calculated values in the central region and near the minima, however, would still remain smaller than the experimental ones by 20%. Considering the simplicity of the model we should not be dissatisfied with this order of agreement.

Because the edges and center of the allowed deuteron recoil momentum favor symmetric excitation of the two intermediate baryons we expect our model to reproduce the maxima of the recoil-momentum spectrum with greater fidelity than the minima. For a beam momentum of 1.88 GeV/*c* the c.m. energy is 2390 MeV which is very close to twice the mass of the Δ(1236) but much removed from twice the mass of the nucleon or of the next baryon excitation the N^*(1470). In the regions of the minima we might expect the contribution from an intermediate N(939) N^*(1470) – which we do not include – to be less negligible though we would be wary of connecting the disparity between theory and experiment in these regions to this specific aspect of our model. The disagreement around the minima is not very large, in any event. In the region of the central maximum the effective ππ mass is about 500 MeV and thus we would expect to find some small enhancement associ-

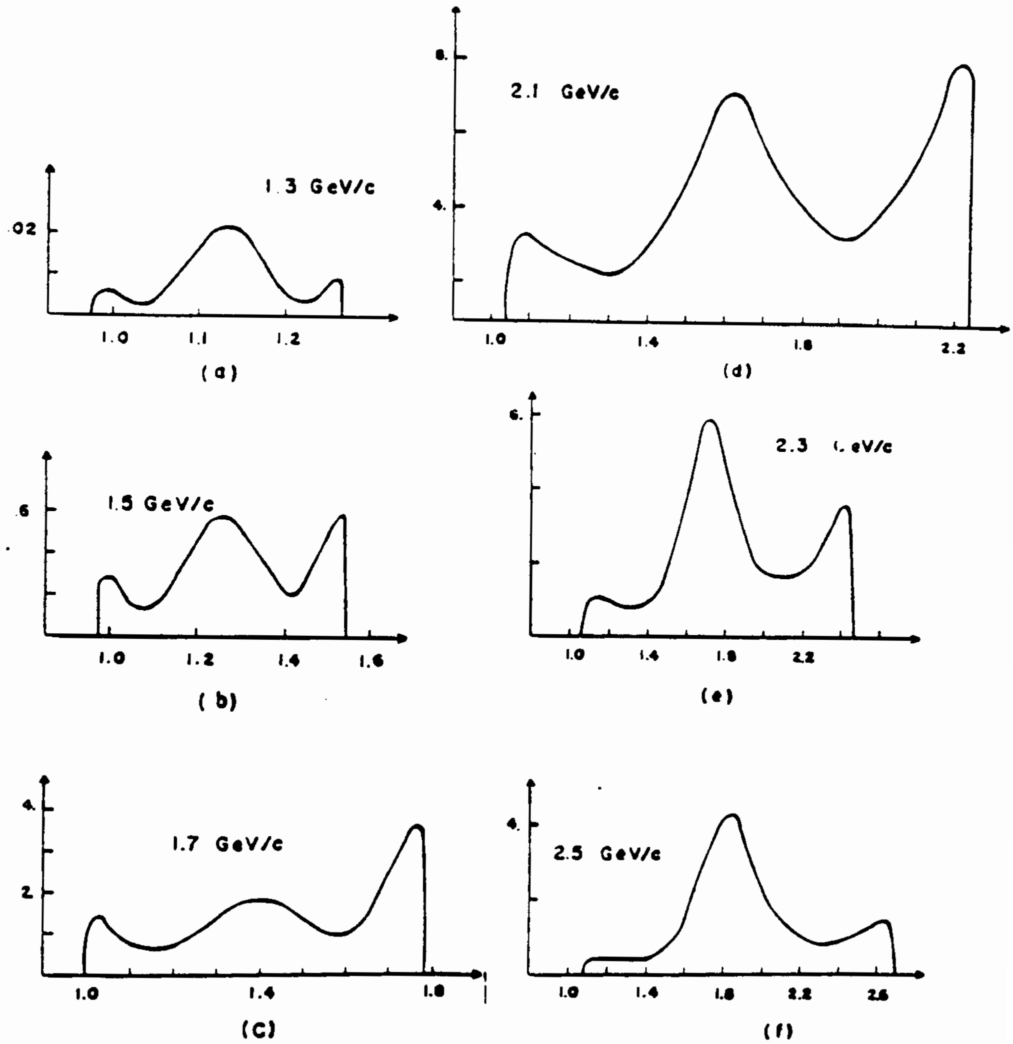

Fig. 4. Predicted differential cross sections for the reaction np → dππ for several different incident lab beam momenta. The ordinates and abscissas are identical to those of fig. 3.

ated with the known ππ interaction at this energy. These would occur largely in the isospin-one channel whose amplitude is sensitive to details of the model owing to the large cancellations mentioned above. For this reason it did not seem appropriate to try to achieve greater agreement in the central region by including an enhancement factor for the final-state interaction between the two pions.

In fig. 4 we have displayed the predicted cross section for the process np → dππ for a number of incident (lab) beam momenta between 1.3 and 2.5 GeV/*c*.

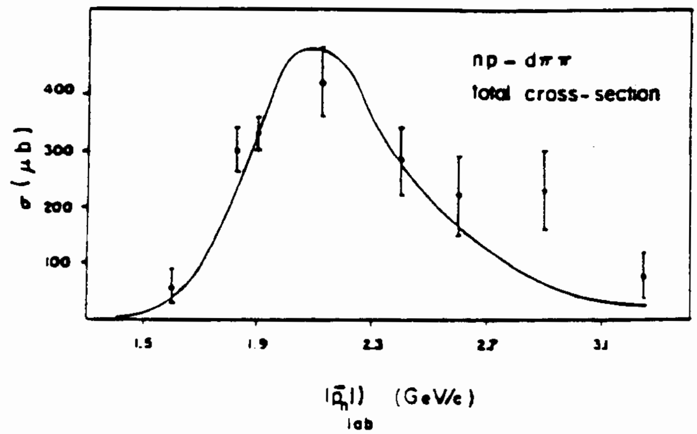

Fig. 5. Comparison of the predicted total cross section for the reaction np → dππ with the Heidelberg-Tel-Aviv data.

The beam momentum 2.1 GeV/c corresponds to a total c.m. energy of $2M_\Delta$, at which energy we might expect the features of our model to be the most strikingly pronounced. We note the general trend with increasing energy that the central peak becomes increasingly more predominant at the expense of the ABC peaks. The predictions for the largest and smallest beam momenta are included for qualitative interest only since the model is not appropriate for c.m. energies far from $2M_\Delta$. We note that the ABC peaks appear even at very low energies.

In fig. 5 we show the total cross section of this reaction for several beam energies. The data are those of the Heidelberg-Tel-Aviv collaboration [9]. In performing the integration over the deuteron solid angle we have assumed the cross section to be roughly isotropic in the c.m. system. This assumption probably leads to no serious errors. The discrepancy between our calculation and experiment at 2.9 GeV/c very likely reflects the contribution of an intermediate $\Delta N^*(1470)$ state.

The calculations of Wolf [5] reproduce the experimental cross sections for the reactions NN → NNππ at the same c.m. energy as the present study. It would seem that more complicated models are necessary in order to reproduce polarization phenomena [11]. Since in the present study we sum over three of the four polarization degrees of freedom it would be surprising should Wolf's parametrization not be adequate for our purposes.

Since Wolf does reproduce the NN → NNππ data we may assume that all contributions to the present process are included implicitly in our calculation if these also contribute to the reaction considered by Wolf. Thus it is unnecessary – it would, in fact, be wrong – to include corrections to our calculation which, supposedly, account for distortion of the initial NN state, the exchange of heavier mesons, an intermediate ΔΔ interaction, or (at our energy) the excitation of other resonances.

In the same way we can eliminate the possibility of an additional contribution

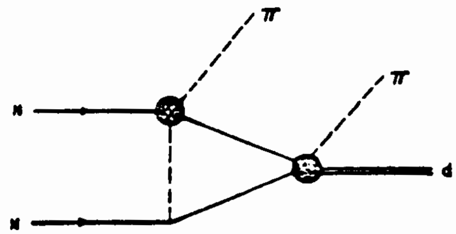

Fig. 6. Possible alternative model for the reaction np → dππ.

from a two-step process as indicated in fig. 6 since this would also be calculated normally as the expectation value between a bound state and a scattering state of the two nucleons of a process occuring between free nucleons.

The mechanism of fig. 6 has been considered as an alternative to one-pion exchange by Bar-Nir et al. [9]. In their treatment of this mechanism and of one-pion exchange the pion, nucleon and deuteron were treated as classical particles whose interactions were described by the experimentally observed cross sections. Since the most important feature of any process between pions and nucleons in this energy region is the excitation of the Δ-resonance it is not surprising that their (qualitative) analyses of the mechanisms of fig. 1 and fig. 6 gave very similar results.

As mentioned earlier we have neglected in our model for the deuteron any contribution from the D-state. Since the largest contribution to the process arises from the small-momentum components of the deuteron wave function, this approximation seems justified here.

There remain two points to consider: (i) the error due to the approximation of eq. (22); and (ii) multiple scattering of the pions.

The first of these can be estimated only by performing a full calculation of the amplitude without this approximation. An attempt at this was made with necessarily very poor statistics. It would seem that this error may be as large as 30% and that the calculated values displayed in fig. 2 and fig. 3 ought perhaps to be reduced by this amount. At the same time we point out that it has been impractical to achieve a statistical accuracy in the Monte-Carlo integration of more than 10 or 15%.

The contribution due to multiple scattering of the pions is difficult to assess since we do not yet possess a theory for calculating these in the region of the 3–3 resonance. Naive application of the Glauber theory leads to corrections of the order of 10% to the impulse approximation for pion-deuteron scattering in the forward direction [12]. The error due to the multiple scattering of the pions is thus rather smaller than those mentioned in the preceding paragraph. To some extent they must also be accounted for in the parametrization of Wolf since πN rescattering can also occur in the reaction NN → NNππ although it will certainly be more important if the momenta of the final nucleons are correlated (as in the deuteron).

In conclusion, it would seem that the OPE model accounts well for all the features of the reaction NN → dππ. The ABC effect is especially well reproduced. One could suggest obvious improvements of the model such as including intermediate nucleons as well as intermediate Δ's or even reggeizing the virtual particles in order to extend the model's range of validity. However, current experimental interest in the reaction does not seem to justify such elaborateness.

The model has an obvious extension to processes involving three and four nucleons. This will be treated in succeeding reports.

The greater part of this work was performed while two of the authors were at other institutions. The first of these (I.B.N.) wishes to thank the Volkswagen Stiftung for its support and the Institut für Hochenergiephysik of the University of Heidelberg for its hospitality during the academic years 1971–1973. The other (M.D.S.) wishes to thank the Institut für Theoretische Kernphysik of the University of Karlsruhe for its support during the academic year 1972–1973. The cooperation of the Saclay Deuteron Group and especially Dr. Lazare Goldzahl in making experimental data available to us and providing helpful comments is gratefully acknowledged.

Appendix A. Isospin structure of the amplitude

The calculation of the isospin matrix element for the processes of fig. 1 is simplified if we regard the isospin factor $\delta_{\alpha\beta}$ in the propagator of the exchanged pion as arising from the decay of a fictitious isoscalar particle into two pions. The same value for the isospin matrix element will be obtained if the reduced matrix element for this vertex is taken to be $\sqrt{3}$.

Using this device the isospin matrix elements for total isospin I may be depicted as in fig. 7, where the dashed, solid, and heavy lines represent particles of isospin 1, $\frac{1}{2}$, and $\frac{3}{2}$, respectively. The calculation of the matrix element reduces to a simple problem of recoupling and the matrix elements are given by

$$C_I = \sqrt{3} \cdot 16\,(2I+1) \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & I \\ 1 & 1 & 0 \\ \frac{3}{2} & \frac{3}{2} & I \end{Bmatrix} \begin{Bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 1 & \frac{1}{2} & \frac{3}{2} \\ I & 0 & I \end{Bmatrix} ,$$

where the quantities in braces are the usual $9-j$ coefficients [6]. Explicit evaluation yields *

$$C_0 = 2/\sqrt{3} , \qquad C_1 = \tfrac{5}{9}\sqrt{2} .$$

Appendix B. Spin summations

In calculating the spin sum of eq. (33) we are led to expressions of the form

* N.b. The values quoted in ref. [1] are too small by a factor $\sqrt{3}$.

Fig. 7. Pictorial representation of the isospin coupling in the reaction NN → dππ.

$$\sum_S \sum_{s_p s_n} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ s_p & s_n & S \end{bmatrix} (\bar{u}(d, s_p) A\, u(p_1, s_1))(\bar{u}(d, s_n) B\, u(p_2, s_2))$$

$$\times \sum_{s'_p s'_n} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ s'_p & s'_n & S \end{bmatrix} (\bar{u}(p_1, s_1) \tilde{C} u(d, s'_p))(\bar{u}(p_2, s_2) \tilde{D}\, u(d, s'_n)) \;.$$

Averaging over the spins of the incoming nucleons leads to the expression

$$\sum_S \sum_{s_p s_n} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ s_p & s_n & S \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ s'_p & s'_n & S \end{bmatrix} X_{s_p s'_p} Y_{s_n s'_n} \,,$$

where the meaning of the matrices X and Y is clear. Explicit evaluation yields

$$\text{spin sum} = [X_{++} Y_{++}] + \tfrac{1}{2} [X_{++} Y_{--} + X_{+-} Y_{-+} + X_{-+} Y_{+-} + X_{--} Y_{++}] + [X_{--} Y_{--}]$$

bracketing separately the contributions for different values of S. With some rearrangement this becomes

$$\begin{aligned} \text{spin sum} &= \tfrac{1}{2} [X_{++} Y_{++} + X_{+-} Y_{-+} + X_{-+} Y_{+-} + X_{--} Y_{--}] \\ &\quad + \tfrac{1}{2} [X_{++} Y_{++} + X_{++} Y_{--} + X_{--} Y_{++} + X_{--} Y_{--}] \\ &= \tfrac{1}{2} \mathrm{Tr}(XY) + \tfrac{1}{2} \mathrm{Tr}(X) \mathrm{Tr}(Y) \,, \end{aligned}$$

which is the expression of eq. (33) above.

Note added in proof

The data of ref. [9], which were interpreted in this analysis as coming from a missing-mass (i.e. counter) experiment, were, in fact, obtained from a bubble-chamber experiment. Thus, these data pertain only to the reaction pn → $d\pi^+\pi^-$ and not also to the reaction pn → $d\pi^0\pi^0$. The magnitude of the calculated total cross sections in fig. 5 should be reduced, therefore, by 10%. This worsens the agreement slightly although it makes it more compatible with the disagreement shown in fig. 3.

References

[1] T. Risser and M.D. Shuster, Phys. Letters 43B (1973) 68 and references therein.
[2] E.A. Remler, Nucl. Phys. B42 (1972) 56 and references therein.
[3] J.D. Bjorken and S.D. Drell, Relativistic quantum mechanics (McGraw-Hill, New York, 1964).
[4] J. Benecke and H.P. Dürr, Nuovo Cimento 56 (1968) 269.
[5] G. Wolf, Phys. Rev. 182 (1969) 1538.
[6] A.R. Edmonds, Angular momentum in quantum mechanics (Princeton University Press, 1957).
[7] G. Bizard et al., Caen-Saclay collaboration, Proc. 5th Int. Conf. on high-energy physics and nuclear structure, Uppsala, Sweden, 1973.
[8] Saclay deuteron group, private communication.
[9] I. Bar-Nir et al., Nucl. Phys. B54 (1973) 17.
[10] J. Banaigs et al., Caen-Saclay collaboration, Proc. 1972 meson Conf., Philadelphia (AIP, New York, 1972).
[11] E. Gotsman and U. Maor, Nucl. Phys. B46 (1972) 525;
G. Fox, Proc. of the Conf. on phenomenology in particle physics, 1971 (Caltech, Pasadena, Calif., 1971).
[12] D. Koltun, Adv. Nucl. Phys., ed. M. Baranger and E. Vogt, vol. 3 (Plenum Press, New York, New York, 1969).