EXOTIC MUON CAPTURE IN NUCLEI AND LEPTON CONSERVATION

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Received 3 August 1972

The process $\mu^- + {}^AZ \to e^+ + {}^A(Z-2)$ is considered in connection with the lepton scheme of Konopinski and Mahmoud. An upper limit for the isotensor weak coupling constant is determined which is smaller than that previously obtained by Kisslinger. The possible connection with recent attempts to discover an isotensor electromagnetic interaction is also discussed.

It has been emphasized by Kisslinger [1] that the study of the reaction

$$\mu^- + {}^AZ \to e^+ + {}^A(Z-2) \quad (1)$$

provided a sensitive test of the lepton-number scheme of Konopinski and Mahmoud [2] in which there is only a single additive lepton number which is conserved and which is +1 for e^-, ν_e, μ^+, ν_μ and −1 for the corresponding antiparticles. The conventional scheme, in which additive muonic and electronic lepton numbers are conserved separately, naturally forbids this reaction as does a third possible scheme combining additive and multiplicative lepton quantum numbers [3].

In addition, Kisslinger assumed for this reaction that there existed "fundamentally" only a pion–lepton isotensor weak coupling (fig. 1) and that the isotensor baryon–lepton couplings arose from this in accordance with the pion-core model [4] of the baryons as shown in fig. 2. This has the interesting consequence that only vector couplings are allowed and hence the isotensor weak and electromagnetic interactions should have a very similar form. However, Kisslinger considered only the ΔΔ–lepton coupling and, therefore, was not led to examine the connection with the electromagnetic results, which apply essentially to the NΔγ vertex. But as will become evident below, it is the NΔ–lepton vertex which likely plays the more important role.

One can, in fact, go somewhat beyond the pion-core model. Assuming with Kisslinger that in the

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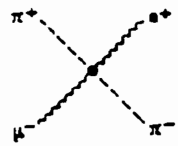

Fig. 1. The weak isotensor pion–lepton vertex consistent with the Konopinski–Mahmoud scheme.

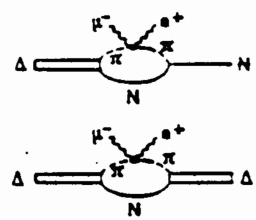

Fig. 2. The weak isotensor baryon-lepton interaction following Kisslinger.

absence of the strong interactions there is only a pion–lepton isotensor coupling of the form

$$L_\pi^{\Delta T=2} = -(\mathrm{i}f/m_\pi)\varphi^- \partial_\mu \varphi^+ L_\mu^* , \quad (2)$$

with $L_\mu = \bar\psi(e^-)\gamma_\mu(1+\gamma_5)\psi(\mu^+)$ leads immediately in perturbation theory to the baryon–lepton interaction which may be expressed by an effective Lagrangian of the form

$$L_B^{\Delta T=2} =$$

$$-\mathrm{i}\frac{G_1}{\sqrt{2}m_\pi}(\bar\psi_\mu\gamma_\nu\gamma_5 I_{-2}^2\psi_N + \bar\psi_N\gamma_\nu\gamma_5 I_{-2}^2\psi_\mu)F_{\mu\nu}^*$$

$$-\mathrm{i}\frac{G_2}{\sqrt{2}m_\pi}\,\bar\psi_\mu I_{-2}^2\psi_\nu F_{\mu\nu}^*$$

$$-\mathrm{i}\frac{G_3}{\sqrt{2}}\bar\psi_\mu\gamma_\nu I_{-2}^2\psi_\mu L_\nu^* + \text{h.c.}, \quad (3)$$

where $F_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu$ and $I^T_{\Delta T}$ is the tensor in isospin space which steps down or steps up the charge appropriately. Further, assuming the dominance of the above graphs we find

$$G_1 \approx \frac{f_{\pi NN}}{f_{\pi N\Delta}}\left[G_2 + \left(\frac{m_\pi}{M_N+M_\Delta}\right)G_3\right]$$

$$\approx 0.44\left[G_2 + \left(\frac{m_\pi}{M_N+M_\Delta}\right)G_3\right], \qquad (4)$$

where $f_{\pi N\Delta} = 2.3$ is the $\pi N\Delta$ (pseudovector) coupling constant and $f_{\pi NN} \equiv (m_\pi/2M_N)g_r = 1.01$ is the pseudovector pion–nucleon coupling constant. Thus, barring delicate cancellations we expect G_1 to be of about the same order of magnitude as the larger of G_2 or $[m_\pi/(M_N+M_\Delta)]\, G_3$. Since G_3 vanishes if we neglect the recoil of the intermediate nucleon we might expect $G_3\,[m_\pi/(M_N+M_\Delta)]$ to be smaller than G_1 or G_2. Even if G_3 should turn out to be numerically larger than G_1 or G_2, this should not be taken as indicating the greater importance of the last term in eq. (3). The other couplings, in fatc, contain an implicit factor (M_N+M_Δ) which compensates that appearing in eq. (4).

The first term of eq. (3) is indeed very similar to the phenomenological $N\Delta\gamma$ interaction of Gourdin and Salin [5] which was used to study pion photoproduction in the (3,3) resonance region:

$$L^{\Delta T=1}_{em} = -ie\frac{g_M}{m_\pi}(\bar{\psi}_\mu\gamma_\nu\gamma_5 I^1_0\psi_N + \bar{\psi}_N\gamma_\nu\gamma_5 I^1_0\psi_\mu)F_{\mu\nu}, \qquad (5)$$

where $F_{\mu\nu}$ is the electromagnetic field tensor. g_M has the numerical value 0.37. In both eqs. (3) and (5) we have neglected the $N\Delta\gamma$ electric coupling since in the isovector electromagnetic interaction its contribution is known to be very small compared to that of the magnetic coupling. In the same way the isotensor electromagnetic coupling is assumed to have the form

$$L^{\Delta T=2}_{em} = -ie\frac{G_T}{m_\pi}(\bar{\psi}_\mu\gamma_\nu\gamma_5 I^2_0\psi_N + \bar{\psi}_N\gamma_\nu\gamma_5 I^2_0\psi_\mu)F_{\mu\nu}. \qquad (6)$$

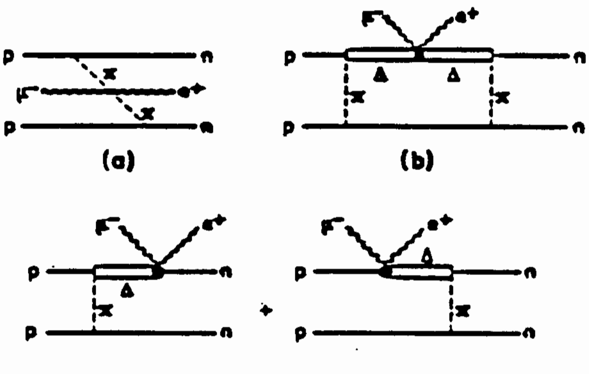

Fig. 3. Possible mechanisms for the reaction $\mu^- + p + p \to e^+ + n + n$.

The Lagrangians of eqs. (2) and (3) lead to three possible mechanisms for the simplest nuclear process

$$\mu^- + p + p \to e^+ + n + n$$

which are shown in fig. 3. These we take to be the basic processes which can occur in nuclei. The amplitude corresponding to fig. 3a vanishes identically between antisymmetric nuclear wave functions. For the remaining two processes we expect that the matrix elements for fig. 3b and 3c behave roughly as

$$M_b \approx |a_\Delta|^2 G',$$

$$M_c \approx 2|a_\Delta| G_1,$$

where G' is some combination of G_2 and G_3 and $|a_\Delta|^2$ is the probability of a virtual Δ being excited in the nucleus, roughly of the order of a few percent. If G_3 is not anomalously large we expect that the amplitude given by fig. 3c will be the larger by an order of magnitude.

The explicit evaluation of the amplitude in fig. 3c leads in the non-relativistic limit to

$$M \approx \tfrac{1}{3}\sqrt{2}G_1 \langle N_f|J_0|N_i\rangle L_0^+, \qquad (7)$$

where $|N_i\rangle$ and $|N_f\rangle$ are the initial and final nuclear states and

$$J_0 = 2C\sum_{k<j} T_j V_j, \qquad (8)$$

with

$$T_{ij} = \exp\{\tfrac{1}{2}i\boldsymbol{k}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)\}\ \tau_i^-\tau_j^- ,$$

$$V_{ij} = V_0(x_{ij})\sigma_i\cdot\sigma_j + V_2(x_{ij})S_{ij} ,$$

$$V_2(x) = (1+3x^{-1}+3x^{-2})\,V_0(x)\,, \qquad V_0(x) = \mathrm{e}^{-x}/x\,,$$

$$x_{ij} \equiv m_\pi|\boldsymbol{x}_i - \boldsymbol{x}_j| ,$$

$$C = \frac{g_r f_{\pi N\Delta}}{18\sqrt{2}\pi}\frac{(M_\Delta+M)m_\pi}{(M_\Delta-M)M_\Delta} \approx 0.29\,, \tag{9}$$

k is the momentum of the emitted positron. We have neglected in eq. (8) terms of order $|k|/4m_\pi \approx 0.1$, among these the space components $\boldsymbol{J}$.

Using the approximation of Riska and Brown [6] – namely, that three-body connected processes may be ignored in eq. (10) below – who applied it to the evaluation of very similar matrix elements appearing in the calculation of the D-state contribution to the pion exchange current, we write

$$J_0 \approx C[V,T]_+ = 2CVT\,, \tag{10}$$

with

$$T = \sum_{i<j} T_{ij}\,, \qquad V = \sum_{i<j} V_{ij}\,.$$

Combining eqs. (7–10) we have for the capture rate in the closure approximation

$$\Lambda(\mu^- \to \mathrm{e}^+) = \frac{|k|^2_{\mathrm{av}}}{2\pi}\,|\varphi_\mu|^2_{\mathrm{av}}\,4C^2\langle N_\mathrm{i}|V^+VT^+T|N_\mathrm{i}\rangle\,, \tag{11}$$

where $|\varphi_\mu|^2_{\mathrm{av}}$ is the muon density averaged over the nuclear volume. Writing approximately

$$\langle N_\mathrm{i}|V^+VT^+T|N_\mathrm{i}\rangle \approx \frac{\langle N_\mathrm{i}|\sum_{ij} V_{ij}^+V_{ij}T_{ij}^+T_{ij}|N_\mathrm{i}\rangle}{\langle N_\mathrm{i}|\sum_{ij} T_{ij}^+T_{ij}|N_\mathrm{i}\rangle}\langle N_\mathrm{i}|T^+T|N_\mathrm{i}\rangle\,. \tag{12}$$

the remainder of the calculation may be performed exactly for the Fermi gas giving

$$\langle N_\mathrm{i}|T^+T|N_\mathrm{i}\rangle = \tfrac{1}{2}Z(Z-1)[\{1-F(\kappa)\}^2 - (Z-1)^{-1}F(\kappa)\{1-F(\kappa)\}]\,, \tag{13}$$

where $\kappa = \frac{1}{2}|k|$ and $F(\kappa)$ is the usual two-body isovector correlation function

$$F(\kappa) = 1 - \tfrac{3}{2}(\kappa/2k_\mathrm{f}) + \tfrac{1}{2}(\kappa/2k_\mathrm{f})^3\,, \quad \kappa < 2k_\mathrm{f}\,,$$
$$= 0\,, \quad \kappa > 2k_\mathrm{f}\,.$$

We have neglected terms in $(N-Z)/A$ since these contribute at most a few percent.

In fact, the relation (13) between the four-body and two-body correlation functions holds to leading order in the proton number Z for an arbitrary determined wave function. In evaluating the right-hand side of eq. (12) we have used the cut-off factor $1-\exp(1.5x_{ij}^2)$ of Riska and Brown [6] to simulate the effects of the hard core.

The rate for ordinary muon capture is given by

$$\Lambda(\mu \to \nu_\mu) = $$
$$= (|\nu|^2/2\pi)|\varphi_\mu|^2_{\mathrm{av}}(G_\mathrm{V}^2 + 3G_\mathrm{A}^2 + G_\mathrm{P}^2 - 2G_\mathrm{A}G_\mathrm{P})Z\{1-F(\nu)\}$$
$$= (|\nu|^2/2\pi)|\varphi_\mu|^2_{\mathrm{av}}(5.4G^2)Z\{1-F(\nu)\}\,, \tag{14}$$

where G_V, G_A and G_P are the effective vector, axial vector and pseudoscalar coupling constants, respectively, $|\nu|$ the average neutrino momentum, and G the Fermi coupling constant ($\approx 10^{-5}/M_\mathrm{N}^2$).

For ordinary muon capture in medium weight nuclei, $|\nu|$ is constrained to have values very near to 80 MeV/c, largely due to the presence of the giant dipole resonance. For the isotensor process, however, the existence of an equally strong 2p–2h collective excitation is not yet evidenced but we may assume that the transition will lead to 2p–2h states in the final nucleus whose strength in centered about 40 MeV. Thus, taking $|k|$ 60 MeV/c and combining eqs. (11)–(14) we obtain for the reaction

$$\mu^- + {}^{65}\mathrm{Cu} \to {}^{65}\mathrm{Co}^* + \mathrm{e}^+\,,$$

$$R \equiv \Lambda(\mu^- \to \mathrm{e}^+)/\Lambda(\mu^- \to \nu_\mu) \approx 0.035\,(G_1/G)^2\,. \tag{15}$$

The factor multiplying $(G_1/G)^2$ is larger than that from Kisslinger's study for his $(G_\Delta/G)^2$ by four orders of magnitude. Two of these simply reflect the trivial difference in the relative strengths of scalar and vector coupling constants. More specifically, if we rewrite the NΔ–lepton interaction [5] as

$$L_{N\Delta I} =$$

$$-iG_4(\bar{\psi}_\mu \gamma_5 I^2_{-2} \psi_N + \frac{1}{M_N + M_\Delta} \bar{\psi}_\mu \gamma_\nu \gamma_5 I^2_{-2} \psi_N \partial_\nu) L^*_\mu$$

$$+ \text{h.c.} + \text{cross term} , \qquad (16)$$

which is identical to first order to the corresponding term in eq. (3) provided $G_4 = \{(M_\Delta + M_N)/m_\pi\} G_I$, then we obtain

$$R \approx 1.4 \times 10^{-4} (G_4/G)^2 . \qquad (17)$$

It is this result which should be compared with Kisslinger's which was essentially

$$R \approx 2.6 \times 10^{-6} (G_3/G)^2$$

assuming the dominance of the process shown in fig. 3b. The remaining two orders of magnitude difference arise from the suppression due to the additional Δ which must be produced as expected from our earlier discussion.

From the experimental upper limit [7] for this branching ratio

$$R_{\text{exp}} \lesssim 2.6 \times 10^{-8} \qquad (18)$$

we infer that

$$G_I \lesssim 0.9 \times 10^{-3} G . \qquad (19)$$

The isotensor interaction and the Konspinski–Mahmoud scheme seem to be largely disfavored.

It is interesting to speculate on the extension to the isotensor coupling of the CVC hypothesis which was proposed only for the *isovector* vector currents. Assuming that a universal scale factor relates all the corresponding weak and electromagnetic couplings we would be led from eqs. (3), (6) and (19) to expect an isotensor electromagnetic coupling

$$g_T \lesssim 0.9 \times 10^{-3} . \qquad (20)$$

Numerous theoretical analysis have been made of possible isotensor interactions in the (3,3) resonance region. Assuming that only the $T = \frac{1}{2}$ final states are important these authors show that

$$M_{\gamma p \to \Delta^+} \approx g_M - \sqrt{\tfrac{2}{3}} g_T ,$$

$$M_{\gamma n \to \Delta^0} \approx g_M + \sqrt{\tfrac{2}{3}} g_T .$$

The most recent experiments [9–11] all measure essentially the quantity $4\sqrt{\frac{2}{3}}(g_T/g_M)$ and find this to be zero within systematic errors of 6%. Using $g_M = 0.37$ we obtain the experimental limit

$$g_T|_{\text{exp}} \lesssim 0.007 \qquad (21)$$

or about eight times larger than the limit suggested in eq. (20) above on the basis of muon capture.

Finally, it is perhaps worth mentioning that if the process (1) were experimentally confirmed, it would be a valuable tool to study the extent to which the Δ manifests itself in the nuclear wave function [12]. The fact that the process proceeds only through the Δ intermediate state makes it quite unique in providing us with information on the role of the nucleon resonances in nuclear physics.

The authors wish to thank Professor L.S. Kisslinger, whose earlier study inspired the present work, for interesting and helpful correspondance.

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